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# Discrete Sampling Analysis for Electricity Market Forecasting with Reproducing Kernel Hilbert Space

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## Abstract

Analyse discrete sampling theories in the reproducing kernel Hilbert space are applied here to whole-sale electricity market forecasting problem. We consider the optimal approximation of any function be longing to the kernel across pricing nodes and hours via a sampling method. Then, a necessary and sufficient condition to perfectly reconstruct the function in the corresponding reproducing kernel Hilbert space of function is investigated. The key idea of our work is adopting the reproducing kernel Hilbert space corresponding to the Gramian matrix of the additive tensor kernel and considering the orthogonal projector by the kernel functions. We also give numerical examples, using the sampling theorem, to confirm the behavior of the proposed method.

Keywords: Gramian matrix, Hilbert space, Orthogonal projector, Reproducing kernel, discrete Sampling theorem.

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## 1. Introduction

Electricity price forecasting has become an important area of research globally since the introduction of the deregulated whole-sale electricity markets. In particular, when compared with other commodities, electricity trade displays a set of attributes that are quite uncommon: constant balance between production and consumption [22]; dependence of the consumption on the time, e.g. hour of the day, day of the week, and time of the year; load and generation that are influenced by external weather conditions [25]; and influence of neighboring markets [14]. Due to these characteristics, the dynamics of competitive electricity markets, electricity price forecasting has become a very valuable tool for all market participants. Producers and consumers can use prediction information to adjust their production schedule and select the best bidding strategy to maximize their respective benefits.

Based on the needs of the energy market, a variety of approaches for electricity price forecasting have been proposed in the last decades, among them, models based on simulation of power system equipment and related cost information [3], game-theory based models which focus on the impact of bidder strategic behavior on electricity prices [26], models based on stochastic modeling of finance [21], regression models [5]

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and artificial intelligence models [15, 13, 11, 12].
In recent years, reproducing kernel Hilbert spaces have been used to solve all kinds of problems in mechanics, astronomy, economical theory, chemical physics, and electrostatics [1, 8, 9, 10]. They have become popular since it is almost universally agreed in the forecasting literature that no single method is best in every situation [16, 24]. Kernel-based function estimation can be also seen from a Bayesian view point. Reproducing kernel Hilbert spaces and linear minimum mean square error function estimators coincide when the pertinent covariance matrix equals the kernel Gram matrix. This equivalence has been leveraged in the context of field estimation, where spatial linear minimum mean square error estimation referred to as Kriging, is tantamount to two-dimensional reproducing kernel Hilbert spaces interpolation [6]. Finally, reproducing kernel Hilbert spaces based function estimators can linked with sampling processes obtained upon defining their tensor kernels [17, 18].
Some recent work uses multiple kernels to build prediction models for electricity load forecasting. For example, in [2], Gaussian kernels with different parameters are applied to learn peak power consumption. In [7], different types of kernels are used for different features and a multi-task learning algorithm is proposed and applied on low level load consumption data to improve the aggregated load forecasting accuracy. However, all of the existing methods rely on a fixed set of coefficients for the kernels, implicitly assuming that all the kernels are equally important for forecasting, which is suboptimal in real world applications.
The purpose of this paper is to introduce an alternative necessary and sufficient condition formula of the kernel-induced sampling theorem specialized for casting electricity price forecasting with uniform sampling on the basis of the theory of Laurent operators by which the difficulty of the inverse of the infinite dimensional Gramian matrix is resolved. Based on the derived results, another proof of the sampling theorem in linear canonical transform domain by the reproducing kernel Hilbert spaces is given. A systematic methodology for judiciously selecting kernels over space and time is the first contribution of this paper. We also investigate the optimal approximation of any band-limited functions in linear canonical transform domain from infinite sampling points associated with results of reproducing kernel Hilbert spaces.
The paper outline is as follows. In Section 2, we give some mathematical definitions about the reproducing kernel Hilbert spaces. Electricity market forecasting is formulated in Section 3, where the novel approach is presented. In Section 4, we will study the optimal approximation by orthogonal projection. In Section 5, we discuss the necessary and sufficient condition to perfectly recover the function in the corresponding reproducing kernel Hilbert space.

## 2. Construction of reproducing kernel Hilbert spaces

In this section, we prepare some mathematical tools concerned with the theory of reproducing kernel Hilbert spaces.

Definition 2.1. Let $X$ be a nonempty set. A function $k : X \times X \longrightarrow \mathbb{R}$ is called a kernel function if there exists a Hilbert space $H$ with an inner product $\langle ., . \rangle_H$ and a map $\varphi : X \longrightarrow X$ such that for any $x$ and $x^{'}$ in the space $X$,

$$k(x, x^{'}) = \langle \varphi(x), \varphi(x^{'}) \rangle_H.$$

Here $\varphi$ is called a feature map, which transforms the data from the input space $X$ to a feature spce $H$, and can be highly complex and even infinite-dimensional.

A function $k : X \times X \longrightarrow \mathbb{R}$ is non-negative definite if for any finite subset $\{x_1, x_2, \dots, x_\ell, \dots\}$ chosen from $X$, the Gram matrix (or kernel matrix) $K = \{k(x_i, x_j)\}_{i,j=1}^{\infty}$ is symmetric and non-negative definite,ie, for any real numbers $a_1, a_2, \dots, a_\ell, \dots,$

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_i a_j k(x_i, x_j) \geqslant 0.$$

Any kernel function $k$ is clearly symmetric and we have

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_i a_j k(x_i, x_j) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \langle a_i \varphi(x_i), a_j \varphi(x_j)\rangle_H = \| \sum_{i=1}^{\infty} a_i \varphi(x_i)\|_H^2 \geqslant 0.$$

Therefore all kernel functions are non-negative definite.

Definition 2.2. Let $H$ be a Hilbert space of real-valued functions defined on a nonempty set $X$. A function $k : X \times X \longrightarrow \mathbb{R}$ is called a reproducing kernel of $H$, and $H$ is a reproducing kernel Hilbert space on $X$, if the following conditions are satisfied:

For any $x \in X$, $k_x(.) = k(., x)$ as a function on $X$ belongs to $H$.

The reproducing property: For any $x \in X$ and any $f \in H$, $\langle f(.), k(., x)\rangle_H = f(x)$. The reproducing property states that the evaluation of $f$ at $x$ can be expressed as an inner product in the feature space. By appling the property, we have, for any $x, x^{'} \in X$

$$k(x, x^{'}) = \langle k(., x), k(., x^{'})\rangle_H.$$

Definition 2.3. Let $H_1$ and $H_2$ be Hilbert spaces. The Schatten product of $g \in H_2$ and $h \in H_1$ is defined by

$$(g \otimes h)f = \langle f, h\rangle_{H_1} g, \quad f \in H_1.$$

Note that $(g \otimes h)$ is a linear operator from $H_1$ onto $H_2$. It is easy to show that the following relations hold for $h, v \in H_1$, $g, u \in H_2$,

$$(h \otimes g)^* = (g \otimes h), \quad (h \otimes g)(u \otimes v) = \langle u, g\rangle_{H_2} (h \otimes v),$$

where the superscript $*$ denotes the adjoint operator.

## 3. Problem formulation

Consider a whole-sale electricity market over a set $S$ of $\mathbb{N}$ commercial pricing nodes indexed by $s$. In a day-ahead market, locational marginal prices correspond to the cost of electricity at each node and over one-hour periods for the following day [19]. Viewing market forecasting as an inference problem, hourly locational marginal prices are the target variables. Energy markets may change significantly due to lasting transmission and generation outages, or shifs in oil or gas markets. That is why the market is considered to be stationary only over the $T$ most recent time periods, which together with the sought next 24 hours comprise the set $T$. The market could be then thought of as a function $Z : S \times T \longrightarrow \mathbb{R}$ to be inferred.
It is postulated that the price at node $s$ time $t$ denoted by $Z(s, t)$ belongs to the function space

$$\mathcal{P} = \{z(s, t) = \sum_{s^{'} \in S, t^{'} \in T} K_{\odot}((s, t), (s^{'}, t^{'})) a_{s^{'}, t^{'}} : \quad a_{s^{'}, t^{'}} \in \mathbb{R}\}, \tag{3.1}$$

defined by $K_{\odot} : (S \times T) \times (S \times T) \longrightarrow \mathbb{R}$. When $K_{\odot}$ is a symmetric positive definite function, the function space $\mathcal{P}$ becomes a reproducing kernel Hilbert space equipped with a finite norm

$$\|Z\|^2_{H_{K_{\odot}}} := \sum_{s, s^{'} \in S} \sum_{t, t^{'} \in T} K_{\odot}((s, t), (s^{'}, t^{'})) a_{s,t} a_{s^{'}, t^{'}}. \tag{3.2}$$

When $K_{\odot}$ is additionally selected as the additive tensor kernel [20], then

$$K_{\odot}((s, t), (s^{'}, t^{'})) := \lambda K_s(s, s^{'}) + (1 - \lambda) K_t(t, t^{'}), \quad 0 \leqslant \lambda \leqslant 1. \tag{3.3}$$

where $K_S : (S \times S) \longrightarrow \mathbb{R}$ and $K_T : (T \times T) \longrightarrow \mathbb{R}$ are kernels over nodes and hours, respectively; then every function in $\mathcal{P}$ can be written as

$$\mathcal{P} = \left\{ z(s,t) = \lambda f(s) + (1-\lambda) g(t) : \quad f \in H_{K_S},\ g \in H_{K_T},\ 0 \leqslant \lambda \leqslant 1 \right\}, \tag{3.4}$$

where $H_{K_s}$ and $H_{K_t}$ are the reproducing kernel Hilbert spaces defined accordingly by $K_S$ and $K_T$, respectively [4]. In both (3.3) and (3.4), the extra coefficient $\lambda$ needs to be determined.
What makes us more interested is the additive tensor kernel. The reason is that very tiny and large values can be produced with the productive tensor kernel, which leads to the Gram matrix closed to representation matrix in practise.

## 4. Kernel specific generalization for electricity market model

In this section, we concentrate on the reproducing kernel Hilbert space $H_{K_\odot}$ corresponding to some reproducing kernel $K_\odot$ as a class of function o which the target functions belong. According to the reproducing property and (3.4), we have

$$z(s_r, t_r) = \lambda f(s_r) + (1-\lambda) g(t_r) = \lambda \langle f(.), K_S(., s_r) \rangle_{H_{K_S}} + (1-\lambda) \langle g(.), K_T(., t_r) \rangle_{H_{K_T}} \tag{4.1}$$

is obtained, where $S$ maps an element of $H_{K_S}$ onto $\mathbb{R}^\infty$ and $T$ maps an element of $H_{K_T}$ onto $\mathbb{R}^\infty$. Let $e_i$ be the unit vector in $\mathbb{R}^\infty$ with only the $i$-th component being unity and let

$$Z = [z(s_1, t_1), z(s_2, t_2), \ldots, z(s_\ell, t_\ell), \ldots]^{\mathcal{T}} \in \mathbb{R}^\infty$$

with $\mathcal{T}$ denoting the transposition operator. Then, applying the Schatten product to equation (4.1) yields

$$Z = \lambda \Big( \sum_{i=1}^{\infty} [e_i \otimes K_S(., s_i)] \Big) f(.) + (1-\lambda) \Big( \sum_{i=1}^{\infty} [e_i \otimes K_T(., t_i)] \Big) g(.). \tag{4.2}$$

For a convenience of description, we write

$$A = \Big( \sum_{i=1}^{\infty} [e_i \otimes K_S(., s_i)] \Big), \quad B = \Big( \sum_{i=1}^{\infty} [e_i \otimes K_T(., t_i)] \Big). \tag{4.3}$$

$A$ is a linear operator defined by $K_S$ and $B$ is a linear operator defined by $K_T$. Then, we have

$$Z = \lambda A f(.) + (1-\lambda) B g(.). \tag{4.4}$$

which represents the sampling process of $f(.) \in H_{K_S}$ and $g(.) \in H_{K_T}$ with the sampling point $S$ and $T$, respectively. Therefore,function reconstruction process can be regarded as an inversion problem for (4.4). Let $\Phi$ be a closed linear subspace in $H_{K_S}$, spanned by the basis functions $\{K_S(., s_i) : \ i \in \mathbb{N}\}$ and $\Psi$ be a closed linear subspace in $H_{K_T}$, spanned by the basis functions $\{K_T(., t_i) : \ i \in \mathbb{N}\}$, i.e.,

$$\Phi = \overline{\mathrm{span}\{K_S(., s_i) : \ i \in \mathbb{N}\}}, \quad \Psi = \overline{\mathrm{span}\{K_T(., t_i) : \ i \in \mathbb{N}\}}, \tag{4.5}$$

then $\Phi^\perp = \mathcal{N}(A)$ and $\Psi^\perp = \mathcal{N}(B)$, where $\mathcal{N}(A)$ and $\mathcal{N}(B)$ denoted the null space of $A$ and $B$, respectively. Any function $f(.) \in \Psi$ and $g(.) \in \Psi$ can be represented by

$$f(.) = \sum_{i=1}^{\infty} \alpha_i K_S(., s_i), \quad g(.) = \sum_{i=1}^{\infty} \beta_i K_T(., t_i),$$

with coefficients $\alpha_i,\ \beta_i \in \mathbb{R}$. For any $z(.,.) = \lambda f(.) + (1-\lambda)g(.) \in \Phi + \Psi$,

$$\begin{aligned}\|z(.,.)\|^2_{H_{K_\odot}} &= \langle \sum_{i=1}^{\infty} \gamma_i K_\odot((.,.),(s_i,t_i)), \sum_{j=1}^{\infty} \gamma_j K_\odot((.,.),(s_j,t_j))\rangle_{H_{K_\odot}} \\ &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \gamma_i\gamma_j \langle K_\odot((.,.),(s_i,t_i)), K_\odot((.,.),(s_j,t_j))\rangle_{H_{K_\odot}} \\ &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \gamma_i\gamma_j K_\odot((s_i,t_i),(s_j,t_j)) \\ &= \lambda \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \alpha_i\alpha_j K_S(s_i,s_j) + (1-\lambda)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \beta_i\beta_j K_T(t_i,t_j) \\ &= \lambda\alpha^{\mathcal{T}} D\alpha + (1-\lambda)\beta^{\mathcal{T}} G\beta < \infty,\end{aligned}$$

holds, where $\alpha = [\alpha_1,\alpha_2,\dots,\alpha_\ell,\dots]^{\mathcal{T}} \in \mathbb{R}^\infty$, $\beta = [\beta_1,\beta_2,\dots,\beta_\ell,\dots]^{\mathcal{T}} \in \mathbb{R}^\infty$, $D = \Big(K_S(s_i,s_j)\Big) \in \mathbb{R}^{\infty\times\infty}$ and $G = \Big(K_T(t_i,t_j)\Big) \in \mathbb{R}^{\infty\times\infty}$ denotes the Gramian matrix of kernels $K_S$ and $K_T$ with sampling points $S, T$, respectively. we intend to use $\Phi$ and $\Psi$ as linear subspaces to which a reconstructed functions belongs. Therefore, since $\Phi$ and $\Psi$ are closed, then

$$H_\alpha = \{\alpha \in \mathbb{R}^\infty :\ \alpha^{\mathcal{T}} D\alpha < \infty\}, \quad H_\beta = \{\beta \in \mathbb{R}^\infty :\ \beta^{\mathcal{T}} D\beta < \infty\}, \tag{4.6}$$

are Hilbert spaces which are homeomorphic with $\Phi$ and $\Psi$, respectively.
It is easy o show that $D$ and $G$ are also reproducing kernels [23]. Thus $D$ and $G$ have the unique corresponding reproducing kernel Hilbert spaces by $H_D$ and $H_G$, respectively. Since $H_D$ and $H_G$ are complete and closed, the exist symmetric and non-negative matrixes $M$ and $N$ that specifies the metric of $H_D$ and $H_G$. Thus, $H_D$ and $H_G$ are characterised as

$$H_D = \{d \in \mathbb{R}^\infty :\ d^{\mathcal{T}} Md < \infty\}, \quad H_G = \{g \in \mathbb{R}^\infty :\ g^{\mathcal{T}} Ng < \infty\}, \tag{4.7}$$

According to the reproducing property,

$$De_i \in H_D, \quad Ge_i \in H_G,$$

holds for any $i \in \mathbb{N}$, which implies that each column of $D$ belongs to $H_D$ and each column of $G$ belongs to $H_G$, therefore

$$d_k = \langle d, De_k\rangle_{H_D} = e_k^{\mathcal{T}} DMd, \quad g_k = \langle g, Ge_k\rangle_{H_G} = e_k^{\mathcal{T}} GNg, \tag{4.8}$$

for any $d = [d_1,d_2,\dots,d_\ell,\dots]^{\mathcal{T}} \in H_D$ and $g = [g_1,g_2,\dots,g_\ell,\dots]^{\mathcal{T}} \in H_G$. The summation premultiplied by $e_k$, with respect to $k$ produces

$$d = DMd, \quad g = GNg, \tag{4.9}$$

for any $d \in H_D$, $g \in H_G$. Therefore, since $De_i \in H_D$ and $Ge_i \in H_G$, then

$$De_i = DMDe_i, \quad Ge_i = GNGe_i,$$

hold for any $i \in \mathbb{N}$ and the summation postmultiplied by $e_i^{\mathcal{T}}$, with respect to $i$ yields

$$D = DMD, \quad G = GNG.$$

The last equation implies that $M$ and $N$ are 1-inverse of $D$ and $G$, respectively. When $D$ and $G$ are Matrixoid, $D = M^{-1}$ and $G = N^{-1}$.

5. Optimal approximation by orthogonal projection

In this section, we discuss the orthogonal projection of the function onto the closed linear subspace spanned by the basis functions corresponding to sampling points. Firstly, it is easy to show that the following lemma and theorem can be obtained by a similar proof as proposed in [23].

Lemma 5.1. $D$ and $G$ are closed linear operators from $H_\alpha$ onto $H_D$ and $H_\beta$ onto $H_G$, respectively.

Theorem 5.2. Let $A = (\sum_{i=1}^{\infty}[e_i \otimes K_S(.,s_i)])$ and $B = (\sum_{i=1}^{\infty}[e_i \otimes K_T(.,t_i)])$, then $A \in \mathcal{L}(H_{K_S}, H_D)$ and $B \in \mathcal{L}(H_{K_T}, H_G)$, where $\mathcal{L}(H_{K_S}, H_D)$ denotes the set of bounded linear operators from $H_{K_S}$ onto $H_D$ and $\mathcal{L}(H_{K_T}, H_G)$ denotes the set of bounded linear operators from $H_{K_T}$ onto $H_G$

According to Theorem 5.2, it immediately follows that:

$$A^* \in \mathcal{L}(H_D, H_{K_S}), \quad B^* \in \mathcal{L}(H_G, H_{K_T}),$$
$$A^*A \in \mathcal{L}(H_{K_S}, H_{K_S}), \quad B^*B \in \mathcal{L}(H_{K_T}, H_{K_T}).$$

Now, we obtain main theorem of this paper as follows.

Theorem 5.3. $P = P_1 + P_2$ is the orthogonal projector onto the closed linear subspace $\Phi + \Psi$ in $H_{K_S} + H_{K_T}$, where $P_1 = A^*A$, $P_2 = B^*B$ and $P(h_1 + h_2) = P_1h_1 + P_2h_2$.

Proof. Let $f(s) = \sum_{i=1}^{\infty} \alpha_i K_S(s, s_i)$ and $g(t) = \sum_{i=1}^{\infty} \beta_i K_T(t, t_i)$ be arbitrary functions in $\Phi$ and $\Psi$, respectively. Then

$$\begin{aligned}
Pz(s,t) &= P(\lambda f(s) + (1-\lambda)g(t)) = \lambda P_1 f(s) + (1-\lambda)P_2 g(t) \\
&= \lambda A^*Af(s) + (1-\lambda)B^*Bg(t) = \lambda A^*D\alpha + (1-\lambda)B^*G\beta \\
&= \lambda\Big(\sum_{i=1}^{\infty} K_S(s,s_i)\otimes e_i\Big)D\alpha + (1-\lambda)\Big(\sum_{i=1}^{\infty} K_T(t,t_i)\otimes e_i\Big)G\beta \\
&= \lambda\sum_{i=1}^{\infty} \alpha^{\mathcal{T}} DMe_i K_S(s,s_i) + (1-\lambda)\sum_{i=1}^{\infty} \beta^{\mathcal{T}} GNe_i K_T(t,t_i) \\
&= \lambda\alpha^{\mathcal{T}} DMK_1 + (1-\lambda)\beta^{\mathcal{T}} GNK_2,
\end{aligned}$$

where

$$K_1 = \sum_{i=1}^{\infty} e_i K_S(s,s_i) = [K_S(s,s_1),\ldots,K_S(s,s_\ell),\ldots]^{\mathcal{T}} \in \mathbb{R}^\infty$$
$$K_2 = \sum_{i=1}^{\infty} e_i K_T(t,t_i) = [K_T(t,t_1),\ldots,K_T(t,t_\ell),\ldots]^{\mathcal{T}} \in \mathbb{R}^\infty.$$

Since $A \in \mathcal{L}(H_{K_S}, H_D)$ and $B \in \mathcal{L}(H_{K_T}, H_G)$ such that $K_S(.,s) \in H_{K_S}$ for any $s \in S$ and $K_T(.,t) \in H_{K_T}$ for any $t \in T$, then

$$AK_S(.,s) = \Big(\sum_{i=1}^{\infty}[e_i \otimes K_S(s,s_i)]\Big)K_S(.,s) = K_1 \in H_D$$
$$BK_T(.,t) = \Big(\sum_{i=1}^{\infty}[e_i \otimes K_T(t,t_i)]\Big)K_T(.,t) = K_2 \in H_G.$$

are followed with any fixed $s \in S$ and $t \in T$. Thus, from (4.9)

$$\begin{aligned}
Pz(s,t) &= \lambda\alpha^{\mathcal{T}} DMK_1 + (1-\lambda)\beta^{\mathcal{T}} GNK_2 \\
&= \lambda\sum_{i=1}^{\infty} \alpha_i K_S(s,s_i) + (1-\lambda)\sum_{i=1}^{\infty} \beta_i K_T(t,t_i) \\
&= \lambda f(s) + (1-\lambda)g(t) = z(s,t),
\end{aligned}$$

are obtained for any $s \in S$ and $t \in T$. On the other hand, for any $f(s) \in \Phi^{\perp}$ and $g(t) \in \Psi^{\perp}$,

$$P_1 f(s) = 0, \quad P_2 g(t) = 0,$$

therefore, $Pz(s,t) = 0$ trivially holds for any $s \in S$ and $t \in T$. Thus, we know that $P = P_1 + P_2 = A^*A + B^*B$ is the orthogonal projector on the closed linear subspace $\Phi + \Psi$. This concludes the proof. □

From the definition of $P$, the closed form of $P$ is written as,

$$\begin{aligned}
Pz(s,t) &= P\Big(\lambda f(.) + (1-\lambda) g(.)\Big) \\
&= \Big(\lambda P_1 f(.) + (1-\lambda) P_2 g(.)\Big) = \lambda A^* A f(.) + (1-\lambda) B^* B g(.) \\
&= \lambda\Big(\sum_{j=1}^{\infty} [K_S(.,s_j) \otimes e_j]\Big)\Big(\sum_{i=1}^{\infty} [e_i \otimes K_S(.,s_i)]\Big) f(.) \\
&+ (1-\lambda)\Big(\sum_{j=1}^{\infty} [K_T(.,t_j) \otimes e_j]\Big)\Big(\sum_{i=1}^{\infty} [e_i \otimes K_T(.,t_i)]\Big) g(.) \\
&= \lambda\Big(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} M_{i,j} [K_S(.,s_i) \otimes K_S(.,s_j)]\Big) f(.) \\
&+ (1-\lambda)\Big(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} N_{i,j} [K_T(.,t_i) \otimes K_T(.,t_j)]\Big) g(.) \\
&= \lambda \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} f(s_j) M_{i,j} K_S(.,s_j) + (1-\lambda) \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} g(t_j) N_{i,j} K_T(.,t_j).
\end{aligned}$$

Now, we obtain a necessary and sufficient condition for a reproducing kernel and a set of sampling points to perfectly reconstruct any function in the corresponding reproducing kernel Hilbert space.

Theorem 5.4. $H_{K_S} = \Phi$ and $H_{K_T} = \Psi$ if and only if

$$\begin{aligned}
K_{\odot}\big((s,t),(s,t)\big) &= \lambda \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} K_S(s,s_j) M_{i,j} K_S(s,s_i) \\
&= (1-\lambda) \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} K_T(t,t_j) N_{i,j} K_T(t,t_i),
\end{aligned} \tag{5.1}$$

holds for any $s \in S$ and $t \in T$.

Proof. Since $K_S(.,s) \in H_{K_S}$ and $K_T(.,t) \in H_{K_T}$ for any $s \in S$ and $t \in T$, so, if $H_{K_S} = \Phi$ and $H_{K_T} = \Psi$ holds , then

$$K_S(.,s) - P_1 K_S(.,s) = 0, \quad K_T(.,t) - P_2 K_T(.,t) = 0. \tag{5.2}$$

Thus

$$\begin{aligned}
K_{\odot}\big((.,.),(s,t)\big) = PK_{\odot}\big((.,.),(s,t)\big) &= \lambda P_1 K_S(.,s) + (1-\lambda) P_2 K_T(.,t) \\
&= \lambda \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} K_S(s,s_j) M_{i,j} K_S(.,s_i) \\
&+ (1-\lambda) \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} K_T(t,t_j) N_{i,j} K_T(.,t_i),
\end{aligned}$$

must hold for any $s \in S$ and $t \in T$ at least. So (5.1) holds. On the other hand, if assume that (5.2) holds, then

$$\begin{aligned}
z(s,t) &= \lambda f(s) + (1-\lambda) g(t) \\
&= \lambda \langle f(.), K_S(.,s)\rangle_{H_{K_S}} + (1-\lambda)\langle g(.), K_T(.,t)\rangle_{H_{K_T}} \\
&= \lambda \langle f(.), P_1 K_S(.,s)\rangle_{H_{K_S}} + (1-\lambda)\langle g(.), P_2 K_T(.,t)\rangle_{H_{K_T}} \\
&= \lambda \langle f(.), P_1^* K_S(.,s)\rangle_{H_{K_S}} + (1-\lambda)\langle g(.), P_2^* K_T(.,t)\rangle_{H_{K_T}} \\
&= \lambda \langle P_1 f(.), K_S(.,s)\rangle_{H_{K_S}} + (1-\lambda)\langle P_2 g(.), K_T(.,t)\rangle_{H_{K_T}} \\
&= \lambda P_1 f(s) + (1-\lambda) P_2 g(t) = \big(P\lambda f(s) + (1-\lambda) g(t)\big) = Pz(s,t)
\end{aligned}$$

is obtained for $f(.) \in H_{K_S}$ and $g(.) \in H_{K_T}$ for any $s \in S$ and $t \in T$, since $P_1$ and $P_2$ are orthogonal projection, which implies $H_{K_S} = \Phi$ and $H_{K_T} = \Psi$. It is easy to show that (5.2) identical to

$$\|K_S(.,s) - P_1 K_S(.,s)\|^2_{H_{K_S}} = 0, \quad \|K_T(.,t) - P_2 K_T(.,t)\|^2_{H_{K_T}} = 0.$$

By applying the Pythagoren theorem and similar method to [23], the above equation can be written as

$$\begin{aligned}
\|K_S(.,s) - P_1 K_S(.,s)\|^2_{H_{K_S}} &= \|K_S(.,s)\|^2_{H_{K_S}} - \|P_1 K_S(.,s)\|^2_{H_{K_S}} \\
&= K_S(s,s) - \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} K_S(s,s_j) M_{i,j} K_S(s,s_i) = 0, \\
\|K_T(.,t) - P_2 K_T(.,t)\|^2_{H_{K_T}} &= \|K_T(.,t)\|^2_{H_{K_T}} - \|P_2 K_T(.,t)\|^2_{H_{K_T}} \\
&= K_t(t,t) - \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} K_T(t,t_j) N_{i,j} K_T(t,t_i) = 0.
\end{aligned}$$

Therefore

$$\begin{aligned}
K_S(s,s) &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} K_S(s,s_j) M_{i,j} K_S(s,s_i), \\
K_T(t,t) &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} K_T(t,t_j) N_{i,j} K_T(t,t_i).
\end{aligned} \tag{5.3}$$

Now, by equation

$$K_{\odot}\big((s,t),(s,t)\big) = \lambda K_S(s,s) + (1-\lambda) K_T(t,t)$$

and equation (5.3), we conclude that the equation (5.1) holds. □

Example 5.5. We will applying the derived results in previous relations to the sampling theories associated with a sinc function which can be written as

$$\begin{aligned}
K_S(s_1,s_2) &= \frac{\sin\pi(s_1 - s_2)}{\pi(s_1 - s_2)}, \quad s_1, s_2 \in \mathbb{R}, \\
K_T(t_1,t_2) &= \frac{\sin\pi(t_1 - t_2)}{\pi(t_1 - t_2)}, \quad t_1, t_2 \in \mathbb{R}.
\end{aligned}$$

It is trivial that

$$K_{\odot}\big((s,t),(s,t)\big) = \lambda K_S(s,s) + (1-\lambda) K_T(t,t) = 1, \quad \forall s,t \in \mathbb{R}.$$

Furthermore, matrixes $M$ and $N$ are reduced to the identity operators. Then the right-hand side of (5.1) reduces to

$$
\begin{aligned}
&\lambda \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} K_S(s,i)M_{i,j}K_S(s,j) + (1-\lambda)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} K_T(t,i)N_{i,j}K_T(t,j) \\
&= \lambda \sum_{i=1}^{\infty}\left(\frac{\sin\pi(s-i)}{\pi(s-i)}\right)^2 + (1-\lambda)\sum_{i=1}^{\infty}\left(\frac{\sin\pi(t-i)}{\pi(t-i)}\right)^2 \\
&= \lambda \sum_{i=1}^{\infty}\left(\frac{(-1)^i\sin\pi s}{\pi(s-i)}\right)^2 + (1-\lambda)\sum_{i=1}^{\infty}\left(\frac{(-1)^i\sin\pi t}{\pi(t-i)}\right)^2 \\
&= \lambda + (1-\lambda) = 1.
\end{aligned}
$$

Thus, it is concluded that (5.1) holds for any $s, t \in \mathbb{R}$.

## 6. Conclusion

In this paper, we discussed discrete sampling theories for Electricity Market Forecasting model. We reformulated a class of machine learning-based as a reproducing kernel Hilbert space and gave a closed-form of the corresponding kernel functions, that is, the difference of ordinary sinc kernels. Moreover, we discussed the kernel-induced sampling theorem for a translation-invariant reproducing kernel Hilbert space corresponding to the additive tensor kernel and introduced an alternative and convenient necessary and sufficient condition formula specialized for these cases.

## References


[1] N. Aronszajn, Theory of reproducing kernels, Trans. Amer. Math. Soc., 68(3) (1950), 337–404. 1

[2] P. Atsawathawichok, P. Teekaput, T. Ploysuwan, Long term peak load forecasting in Thailand using multiple kernel Gaussian Process, In ECTI-CON, (2014), 1–4. 1

[3] J. Bastian, J. Zhu, V. Banunarayanan, R. Mukerji, Forecasting energy prices in a competitive market, IEEE computer applications in power 12(3) (1999), 40–45. 1

[4] C. Berg, J. P. R. Christensen, P. Ressel, Harmonic analysis on semigroups, Springer-Verlag, New-York, 1984. 3

[5] J. Contreras, R. Espínola, F. J. Nogales, A. J. Conejo, ARIMA models to predict next-day electricity prices, IEEE Trans. Power Syst., 18 (2003), 1014–1020. 1

[6] N. Cressie, Statistics for Spatial Data, Wiley, 1991. 1

[7] J. B. Fiot, F. Dinuzzo, Electricity demand forecasting by multi-task learning, IEEE Trans. Smart Grid, 99 (2016). 1

[8] M. R. Foroutan, R. Asadi, A. Ebadian, A reproducing kernel Hilbert space method for solving the nonlinear three-point boundary value problems, International Journal of Numerical Modelling: Electronic Networks, Devices and Fields, 32(3) (2019), 1–18. 1

[9] M. R. Foroutan, A. Ebadian, R. Asadi, Reproducing kernel method in Hilbert spaces for solving the linear and nonlinear four-point boundary value problems, International Journal of Computer Mathematics, 95(10) (2018), 2128–2142. 1

[10] M. R. Foroutan, A. Ebadian, H. Rahmani Fazli, Generalized Jacobi reproducing kernel method in Hilbert spaces for solving the black-scholes option pricing problem arising in financial modeling, Mathematical Modelling and Analysis, 23(4) (2018), 538–553. 1

[11] P. S. Georgilakis, Artificial intelligence solution to electricity price forecasting problem, Appl. Artif. Intell., 21 (2007), 707–727. 1

[12] P. S. Georgilakis, Market Clearing Price Forecasting in Deregulated Electricity Markets Using Adaptively Trained Neural Networks, In Proceedings of the fourth Helenic Conference on Artificial Intelligence, Heraklion, Greece, 18-20 (2006), 56–66. 1

[13] Y. Hong, C. P. Wu, Day-ahead electricity price forecasting using a hybrid principal component analysis network, Energies, 5 (2012), 4711–4725. 1

[14] J. Lago, F. De Ridder, P. Vrancx, B. De Schutter, Forecasting day-ahead electricity prices in Europe: the importance of considering market integration, Appl. Energy, 211 (2018), 890–903. 1

[15] P. Mandal, A. K. Srivastava, T. Senjyu, M. Negnevitsky, A new recursive neural network algorithm to forecast electricity price for PJM day-ahead market, Int. J. Energy Res., 34 (2010), 507–522. 1

[16] M. Z. Nashed, Q. Sun , Function spaces for sampling expansions, Multiscale Signal Analysis and Modelling, Lecture Notes in EE, Springer (2012), 81–104. 1

[17] M. Z. Nashed, Q. Sun, Sampling and reconstruction of signals in a reproducing kernel subspace of LpRd, J. Funct. Anal., 258(7) (2010), 2422–2452. 1

[18] M. Z. Nashed, G. G. Walter, General sampling theorem for functions in reproducing kernel Hilbert space, Math. Control, Signals Syst., 4(4) (1991), 363–390. 1

[19] A. L. Ott, Experience with PJM market operation, system design, and implementation, IEEE Trans. Power Syst., 18(2) (2003), 528–534. 3

[20] Y. Pan, M. Jiang, LRR-TTK DL for face recognition, IET Biometrics, 6(3) (2017), 165. 3

[21] S. Schneider, Power spot price models with negative prices, J. Energy Mark, 4 (2012), 77–102. 1

[22] M. Shahidehpour, H. Yamin, Z. Li, Market overview in electric power systems. Market operations in electric power systems, New York (USA): John Wiley and Sons, Inc., 2 (2002), 1–20. 1

[23] A. Tanaka, H. Imai, M. Miyakoshi, Kernel-induced sampling theorem, IEEE Trans. Signal Process, 58(7) (2009), 3569–-3577. 4, 5, 5

[24] G. Wahba, Spline Models for Observational Data, Society for Industrial and Applied Mathematics, PA 1990. 1

[25] R. Weron, Electricity price forecasting: a review of the state-of-the-art with a look into the future, Int. J. Forecast, 30(4) (2014), 1030–1081. 1

[26] H. Zhang, F. Gao, J. Wu, K. Liu, X. Liu, Optimal bidding strategies for wind power producers in the day-ahead electricity market, Energies, 5 (2012), 4804–4823. 1